Squares

Submission deadline: September 30^{th} 2018

Find the numbers in the sequence

$$11, 111, 1111, 11111, \cdots$$

that are squares of integers.

The problem was solved by

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Discussion:

The given numbers are of the form $1 + 10 + 10^2 + \cdots + 10^n$. Suppose that $1 + 10 + 10^2 + \cdots + 10^n = m^2$. Notice that $1 + 10 + 10^2 + \cdots + 10^n$ is an odd integer therefore m must be an odd integer as well. Thus

$$1 + 10 + 10^2 + \cdots + 10^n = (2p + 1)^2$$

Which results in

$$10(1 + 10 + \cdots + 10^{n-1}) = 4p(p + 1).$$

Thus

$$5(1 + 10 + \cdots + 10^{n-1}) = 2p(p + 1)$$

In the equality above, the number on the left hand side is an odd integer while the number on the right hand side is an even integer. Thus none of the numbers is a square of an integer.